

The analysis of sire buying policies

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Summary

An expression is derived for the net present value of returns obtained through increased productivity of descendants of a purchased sire. This expression is useful for evaluating a given sire purchase, but is not so helpful in determining a sire buying policy. For this purpose, an analysis is made of the value of sire purchase from a breeding nucleus which is making constant genetic gains when the lag of the commercial population behind the nucleus has stabilised under a fixed policy. This leads to a criterion for deciding how long a sire should be used, which depends on price, breeding value, and rate of genetic gain in the nucleus. The optimum sire buying policy depends on the relation between price and breeding value of the sire. Competition for sires may change the relationship of price to breeding value so that no class of sire is the best buy if properly used and it is shown that in this case price is a quadratic function of breeding value.

Prices paid by producers for boars auctioned from French performance test stations were found to be related to index value (I) by the approximate equation :

$$\text{price (francs)} = 1\,500 + 5(I - 100) + 0.5(I - 100)^2.$$

For the observed price function the best policy is to buy boars with high (about 140) index value, and replace them after 15 months use. The net present value of an average boar is estimated as 2 000 francs.

Introduction

Most economic assessments of animal breeding programs, such as those of POUTOUS and VISSAC (1962), HILL (1971), JAMES (1972) and ELSÉN and MOCQUOT (1974) have dealt with decisions made in the breeding nucleus. However, genetic gains made in the nucleus are mostly realised by gene flow into commercial populations and, therefore, the benefit of genetic gain is dependent on decisions made

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by owners of commercial stock to buy breeding animals, normally sires, from the breeding nucleus. Thus genetic gains made in the nucleus will have a significant impact on production to the extent that purchase of sires from the nucleus is seen as a profitable use of resources by owners of production stock. NAPIER and JONES (1976) have discussed the value of Australian merino rams by calculating the discounted value of returns from the progeny of a purchased ram. However, they ignored contributions to later generations, and considered evaluation of a particular sire, rather than a sire buying policy. Their approach may thus be more relevant to buying a terminal sire than to purchase of a sire for purebreeding. In this paper, the analysis of sire buying policies for purebred populations will be developed in relation to the rate of genetic gain being achieved in the nucleus, and to the relative breeding value and price of available sires.

Evaluation of a purchased sire

We first develop an expression for the value of a purchased sire, given all the conditions of use. This will be expressed as present value of discounted returns, comparing the purchased sire with a randomly chosen male from the commercial population or base. The notation to be used in this paper is listed with definitions in the Appendix.

N sires are used altogether at any one time, a fraction W of them surviving to the next time unit. For simplicity, survival rate is assumed independent of age, and time units are referred to as periods. The maximum number of times a sire will be used is T , provided he survives long enough. If n replacement sires are bought each period :

$$N = n + nW + \dots + nW^{T-1} \\ = n(1 - W^T)/(1 - W) \text{ or } nT \text{ if } W = 1$$

Thus, if C is the purchase cost per sire, the expense of sire buying each period is $NC(1 - W)/(1 - W^T)$ or NC/T if there is no wastage.

If f_j , $j = 1, 2, \dots, F$, are the genetic contributions of females of age j to the progeny crop, where $\sum_j f_j = 0.5$, and each sire contributes an equal number of offspring in any period, it can be shown that the genetic contribution of a sire to the progeny crop at time t from its initial use is $e' F^{t-1} e/2N$, where

$$e' = (1 \quad 0 \quad \dots \quad 0 \quad 0) \\ \text{and} \quad \underline{F} = \begin{bmatrix} f_1 & f_2 & \dots & f_{F-1} & f_F \\ 1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

Using a discount rate d , the contribution at time t is multiplied by r^t to bring it to present value, where $r = 1/(1 + d)$, and summing these discounted values over all values of t gives $re' (\underline{I} - r\underline{F})^{-1} e/2N$. Letting J denote $e' (\underline{I} - r\underline{F})^{-1} e$, it can be shown that :

$$J = 1/(1 - \sum_j f_j r^j)$$

The sire will make a similar contribution at each subsequent mating in which he is used, and these can be combined after discounting by a further factor of

r for each time unit. Allowing for wastage, the expected total genetic contribution of a sire is :

$$(rJ / 2N (1 - r^T W^T) / (1 - rW))$$

If it is expected that NP progeny will be produced each period, the number of discounted progeny genotypes expected from a sire is :

$$\frac{1}{2} r P J (1 - r^T W^T) / (1 - rW)$$

We now suppose returns from progeny are realised at an age Y units in both sexes. There is no real loss of generality in this assumption since, if returns occur over several periods, as in wool sheep, these can all be discounted back to the same period and summed (HOPKINS and JAMES, 1979). If each unit of breeding value is worth B economic units per animal, the value of a unit of breeding value in the sire is :

$$\frac{1}{2} r^{Y+1} B P J (1 - r^T W^T) / (1 - rW)$$

This remains true when $W = 1$. Since the sire cost is C , if he has a breeding value R relative to the mean of the base, his net present value is :

$$\frac{1}{2} r^{Y+1} B R P J (1 - r^T W^T) / (1 - rW) - C \quad (1)$$

It is obvious from (1) that the larger T is the greater is the net present value of the sire, as was noted by NAPIER and JONES (1976) in their example, and as is clear from the fact that a sire contributes more descendants the longer he is used. However, the problem with trying to define an optimum sire buying policy on the basis of this expression is that the comparison of breeding value is made with contemporary base bred males. If the nucleus is making genetic gains there will be young sires available for purchase which are genetically superior to the sires already in use. It is then likely that at some value of T the genetic superiority of young nucleus males over the oldest sires in use will be great enough to warrant the buying of these young sires as replacements for the old ones. In essence what is missing from the above treatment is a recognition of the cost of not replacing old sires by better young ones. The stimulus to the nucleus to produce genetic improvement is precisely the fact that if it does not do so it is in the interest of buyers to restrict their purchases of sires as far as possible.

Consideration of lag costs

One of the difficulties with the method outlined in the previous section is that the consequences of a single decision (sire purchase) over a long time period are taken into account under the assumption that subsequent actions are fixed. If we allow for possible variation in subsequent actions, then the value of present actions will change. For example, the effect of selection made at a given time depends on the generation length after selection has been made. To evaluate a policy we should consider an average over many periods. The simplest approach is to consider an equilibrium situation. If a producer continually buys sires from a nucleus making constant genetic gains, his population will eventually be improving at the same rate as the nucleus, but will lag behind by an amount depending on generation length and selection differentials (BICHARD, 1971). The mean productivity of one population relative to another will depend on the diffe-

rence between their lags, so that the lag of a population behind the nucleus should be a good measure of its profitability.

If G is the rate of genetic gain per time unit in the nucleus, C_B is the genetic selection differential applied to breeding animals in the base, where the generation length is l_B , the genetic lag A can be written, as shown by JAMES (1977), as :

$$A = 2(l_B G - C_B)$$

If D_{MB} and d_{FB} are genetic selection differentials applied to nucleus born males and base born females for use in the base;

$$C_B = \frac{1}{2}(D_{MB} + d_{FB})$$

and if l_{MB} and l_{FB} are the average ages of male and female parents used in the base :

$$l_B = \frac{1}{2}(l_{MB} + l_{FB}) \quad (2)$$

Therefore : $A = (l_{MB}G - D_{MB}) + (l_{FB}G - d_{FB})$

If there were no lag at all, the returns from the base in each period would be increased by an amount NPBA, which we may therefore call the « lag cost ». The lag cost may then be written as :

$$NPB [(l_{MB}G - D_{MB}) + (l_{FB}G - d_{FB})]$$

The value of l_{MB} depends on the sire replacement rate. If there is no wastage and sires have equal numbers of progeny at ages $a + 1$, $a + 2$, ... $a + T$, l_{MB} is $a + \frac{1}{2}(T + 1)$. If the survival rate is W per time unit, then :

$$l_{MB} = a + \frac{1}{1 - W} - \frac{TW^T}{1 - W^T}$$

The cost of sires per time unit is nC where $n = N/T$ if $W = 1$ and $n = N(1 - W)/(1 - W^T)$ if $W \neq 1$. The total cost per period, considering both lag cost and sire purchase cost, is :

$$NPB[(l_{MB}G - D_{MB}) + (l_{FB}G - d_{FB})] + nC$$

The second of the three terms in this expression is independent of sire buying policy, so the variable cost is :

$$\begin{aligned} V &= NPB(l_{MB}G - D_{MB}) + nC \\ &= NPB \left[G \left(a + \frac{1}{1 - W} - \frac{TW^T}{1 - W^T} \right) - D_{MB} \right] + \frac{NC(1 - W)}{1 - W^T} \end{aligned} \quad (3)$$

or when $W = 1$:

$$V = NPB \left[G \left(a + \frac{T + 1}{2} \right) - D_{MB} \right] + NC/T \quad (4)$$

We shall suppose that sire cost, C , and sire selection differential, D_{MB} , are fixed and find the value of T which minimises the variable cost. That is, we seek an optimum replacement rate for sires of a given type. We later take up the question of sire buying policies when cost and breeding value of sires are variable.

It is convenient to express the cost in units of NPBG, the increase in returns per time unit, and to express the sire selection differential as S , measured in units

of rate of genetic gain, where $S = D_{MB}/G$. Also, the cost of a sire relative to the increase in returns per sire per period will be denoted $Q = C/PBG$. With this notation :

$$U = \frac{V}{NPBG} = a + \frac{I}{I - W} - \frac{TW^T}{I - W^T} - S + \frac{Q(I - W)}{I - W^T}$$

or, when $W = I$,

$$U = a + \frac{1}{2}(T + I) - S + Q/T$$

We then find :

$$\begin{aligned} \frac{dU}{dT} &= \frac{W^T}{(I - W^T)^2} [Q(I - W) \ln W - I + W^T - T \ln W], \quad W \neq I \\ &= \frac{1}{2} - Q/T^2, \quad W = I. \end{aligned}$$

For any given values of W and Q , there is an optimum number of periods of use of a sire which minimises total cost. This number is the solution of

$$\begin{aligned} W^T - T \ln W &= I - Q(I - W) \ln W, \quad W \neq I \\ T^2 &= 2Q, \quad W = I. \end{aligned}$$

The second expression is the limiting form of the first as $W \rightarrow I$, and shows that as Q increases, that is, sires become more expensive relative to the genetic benefits they confer, fewer sires should be bought each period. The result when there is wastage is not so obvious from the equation, but numerical results are given in table I, showing the optimum T values for a range of values of Q and W .

TABLE I

*Optimum number of times (T) a sire is used
Le nombre optimal de fois (T) qu'un père est utilisé*

Survival rate (W) Taux de survie (W)	Relative cost of sire a (Q) Prix relatif d'un père (Q)						
	0.5	1	2	4	8	16	32
1	1	1.4	2	2.8	4	5.7	8
0.95	1.0	1.4	2.0	2.9	4.1	5.9	8.5
0.9	1.0	1.4	2.0	2.9	4.2	6.1	9.0
0.75	1.0	1.4	2.0	3.0	4.5	7.0	11.3

In general, solutions are not integers, but this is not important, since $T = 4.5$ can be interpreted as using half the sires 4 times and half 5 times, though wastage makes this a little more complicated. When Q is small, so that sires are cheap relative to the rate of genetic improvement, sires should be turned over quickly, and the optimum is not much affected by the wastage rate, essentially because the rapid turnover does not allow time enough for wastage to produce effects. But

if sires are expensive and should be used often, a high wastage rate means that there are many fewer older sires than when wastage rate is low, and thus the generation length and lag do not rise so rapidly with T . Therefore sires should be used more often if possible when wastage rates are high than when they are low. For example, when $Q = 32$, a sire is used 8 times if $W = 1$, but up to 11.3 times if $W = 0.75$. However, the average age of sires used is lower ($a + 3.55$) when $W = 0.75$ than when $W = 1(a + 4.5)$.

Variation in price and breeding value of sires

In the previous section we considered costs for sires of given price and breeding value. Except when sires were to be used very many times, wastage rate had little effect on optimum policy. In this section, to avoid mathematical complications, it will be assumed there is no wastage of sires. However, it will be assumed that price and breeding value are variable. The price of a sire is related to his breeding value by the function :

$$C = PBG Q_0 k(S)$$

where $k(0) = 1$, so that Q_0 is the value of Q for a sire of average breeding value. The variable cost equation may then be written as :

$$U = a + \frac{1}{2}(T + 1) - S + Q_0 k(S) / T$$

We can now consider variation in both S and T . First,

$$\frac{\partial U}{\partial T} = \frac{1}{2} - Q_0 k(S) / T^2$$

which is the result of the previous section with Q replaced by $Q_0 k(S)$. Assuming $k(S)$ is an increasing function of S , this implies better sires should be used longer than worse ones — hardly a surprising result.

Second,

$$\frac{\partial U}{\partial S} = -1 + \frac{Q_0}{T} \frac{\partial k}{\partial S}$$

The effect of variation in S depends on the nature of the function $k(S)$. The simplest assumption is that $k(S)$ is a linear function :

$$k(S) = 1 + bS$$

from which

$$\frac{\partial U}{\partial S} = -1 + \frac{Q_0 b}{T}$$

Therefore if $T > Q_0 b$, U decreases as S increases, while if $T < Q_0 b$, U increases as S increases. Thus for any given value of T it is best to have S either as large or as small as possible. There is no overall minimum, in the mathematical sense, with this form of $k(S)$. The equations :

$$\frac{\partial U}{\partial T} = \frac{\partial U}{\partial S} = 0$$

define a saddle point, not a minimum. Another mathematical difficulty is that with a linear function the price of very poor sires will be negative, the nucleus

paying the base to use very poor sires. This seems unrealistic, but if the slope of the line is not too great negative prices will occur only for values of S outside the range of the population and thus are irrelevant.

A numerical illustration of the relationship of U to S and T when $k(S)$ is linear is shown in table 2, where $a = 1$, $Q_0 = 20$ and $b = 0.1$. It is assumed that S may vary over the range ± 10 , corresponding to the top and bottom 5 per cent of nucleus males if the coefficient of variation of sire breeding values is 10 per cent and genetic gains are being made at the rate of 2 per cent per period. The value of b then corresponds to the top 5 per cent of sires having twice the average price, while the bottom 5 per cent have zero price. In this case the solution of

$$\frac{\partial U}{\partial T} = \frac{\partial U}{\partial S} = 0$$

is given by $T = 2$, $S = -9$.

TABLE 2

Relative total cost (U) as a function of S and T
Le coût global relatif (U) en fonction de S et T
 $Q = 20 + 2 S$

Value of S Valeur de S	Value of T Valeur de T							Optimum T T optimal
	1	2	3	4	6	8	10	
10	32	12.5	6.3	3.5	1.2	0.5	0.5	8.9
5	27	12.5	8.0	6.0	4.5	4.3	4.5	7.7
0	22	12.5	9.7	8.5	7.8	8.0	8.5	6.3
-5	17	12.5	11.3	11.0	11.2	11.8	12.5	4.5
-9	13	12.5	12.7	13.0	13.8	14.8	15.7	2
-10	12	12.5	13.0	13.5	14.5	15.5	16.5	0

The table shows clearly that $T = 2$ marks the point at which U does not depend on S, and also that $T = 2$, $S = -9$ is a saddle point. It is also clear that the optimum value of T increases with S, the value of U at this optimum being lower as S increases. Under these conditions the best policy is to buy the best available sires and use them many times, the total number of times of use being given by the result of the previous section.

However, given this information, one would expect buyers to prefer sires with high breeding values, and the resulting demand would be expected to change the nature of the function $k(S)$. In fact, one might expect that if there is a best policy, in terms of S and T, and this is known to buyers, there would be a greater demand for sires with the optimum value of S, with the result that the price curve $k(S)$ would be pushed into a form in which there was no optimum value of S. Returning to the partial derivatives, we see that for any S, buyers should use

such sires T times where $T^2 = 2Q_0k(S)$. We also want $\frac{\partial U}{\partial S}$ to be zero so that it is possible to compensate for buying a poorer sire by getting it cheaply enough. The condition that $\frac{\partial U}{\partial S} = 0$ is :

$$\frac{\partial k}{\partial S} = \frac{T}{Q_0}$$

and making use of the relation between T and S :

$$\frac{\partial k}{\partial S} = \sqrt{2k(S)/Q_0}$$

The solution of this differential equation under the condition $k(0) = 1$ is given by :

$$k(S) = (1 + S/\sqrt{2Q_0})^2 \quad (5)$$

Then we find :

$$U = a + \frac{1}{2}(T + 1) - S + (S + \sqrt{2Q_0})^2/2T$$

from which :

$$\frac{\partial U}{\partial T} = \frac{1}{2} \left[1 - \left(\frac{S + \sqrt{2Q_0}}{T} \right)^2 \right]$$

$$\frac{\partial U}{\partial S} = -1 + \frac{S + \sqrt{2Q_0}}{T}$$

Both of these derivatives are zero when $T = S + \sqrt{2Q_0}$, and with this relationship between S and T :

$$U = a + \frac{1}{2} + \sqrt{2Q_0}$$

TABLE 3

Relative total cost (U) as a function of S and T
Le coût global relatif en fonction de S et T
 $Q = 0.5(S + 2)^2$

Value of S Valeur de S	Value of T Valeur de T							
	1	2	3	4	6	8	10	12
10	64.0	28.5	17.0	11.5	6.5	4.5	3.7	3.5
8	44.0	19.5	11.7	8.0	4.8	3.8	3.5	3.7
6	28.0	12.5	7.7	5.5	3.8	3.5	3.7	4.2
4	16.0	7.5	5.0	4.0	3.5	3.8	4.3	5.0
2	8.0	4.5	3.7	3.5	3.8	4.5	5.3	6.2
0	4.0	3.5	3.7	4.0	4.8	5.8	6.7	7.7
-1	3.5	3.8	4.2	4.6	5.6	6.6	7.6	8.5

It is to be noted that $k(S)$ has a minimum at $S = -\sqrt{2Q_0}$, being then zero, implying that below this point the price of sires increases as their breeding values fall. However, in the present context this is irrelevant since such sires will never be bought. In fact, since the minimum use of sires is one period, the corresponding value of S is $1 - \sqrt{2Q_0}$, for which $C = \frac{1}{2}PBG$. The zero price corresponds to $T = 0$ and thus is irrelevant. The relationship between U , S and T when $k(S)$ has the equilibrium quadratic form is shown table 3 for $a = 1$, $Q_0 = 2$.

It is noticeable that with this price function, good sires are used very many times because of their high cost. However, the minimum overall cost is the same, 3.5 NPBG, for all classes of sire provided the sires are used the correct number of times, $S + 2$. With this quadratic price function, the total cost can be divided into a lag cost of NPBG $\left(a + \frac{T + 1}{2} - S\right)$ and a sire cost of NC/T which equals :

$$NPBG Q_0 k(S) / T = NPBG (S + \sqrt{2Q_0})^2 / 2T$$

When the optimum policy is used, $T = S + \sqrt{2Q_0}$ and the sire purchase cost is $\frac{1}{2} NPBG (S + \sqrt{2Q_0})$. Thus the purchase costs are still higher for purchasers of genetically superior sires although fewer are bought. On the other hand lag costs are $NPBG (a + \sqrt{Q_0}/2 - \frac{1}{2} S)$, and are smaller when genetically superior sires are purchased. The increase in generation length caused by longer use of expensive sires is not enough to negate the benefits of their genetic superiority, so that the lag would be smaller in those populations which used very good sires but kept them for longer periods.

In this discussion it has been assumed that buyers compete, each knowing the optimum strategy and each having the same breeding objective. There is then no economic advantage in buying sires of any particular quality, provided they meet the minimum requirement $S > 1 - \sqrt{2Q_0}$, and that sires are used an appropriate number of times. On the other hand, if the price function differs from this ideal form, there will be some class or classes of sires which will be more profitable to buy, and such classes can be identified using the type of analysis used here, as was done for a linear $k(S)$.

Prices of boars in France

As seen above, the best policy for sire buyers depends on the form of the price function. As an example of a price function, prices of boars sold at testing stations in France in the year 1977/78 were investigated. For these sales, the data available were the breed of boar (Large White or French Landrace), the price paid, and the relative index values. Relative index value is calculated as an estimate of breeding value scaled to have an average of 100 and a standard deviation of 20 points. In addition, for each boar there was recorded the type of buyer (breeder, A.I. centre or producer). Prices paid by breeders and A.I. centres are not relevant to the present discussion and have been excluded from this study. Excluded boars tended to have high index values, and though there was little comparative information, may have been somewhat more expensive than boars of similar index values bought by producers. Altogether, data were available

for 61 sales held at 12 testing stations, giving prices for 697 Large White and 552 French Landrace boars. Since all boars with an index value below 100 are slaughtered, the available data are for boars with index values of 100 or more. There are few boars with index values above 140, and most of these are bought by breeders or A.I. centres so our attention is concentrated on index values from 100 to 140.

For any one index value, prices showed a considerable range, and there were occasional very high prices. There also appeared on inspection to be some differences between sales in prices paid for boars of the same index value, but since our present purpose is not to make a detailed analysis of boar prices the data were pooled over all 61 sales. Then the median price paid for boars of a given index value (100, 101, 102, ... 140) was determined and plotted for each breed. The median was preferred to the mean because it is less affected by occasional very high prices. The plot is shown in figure 1, and the relation between median price and index value can be seen to be very similar in the two breeds. The curves are erratic near index values of 140. This is mainly because the medians are based on very small numbers. However, since a high proportion of boars in this region are bought by A.I. centres and breeders, this may affect the prices paid by producers. In both breeds, price changes little as index values rise from 100 to about 110, but then rises more rapidly. It has not been thought worthwhile to make a detailed statistical analysis of these prices. Rather, a simple quadratic equation which can describe both relationships without doing excessive violence to the data is plotted in figure 1. The equation of the curve is :

$$\text{price} = 1500 + 5(I - 100) + 0.5(I - 100)^2 \quad (6)$$

Clearly this simple equation describes the price-index relation reasonably well.

There is evidence (P. SELIER and L. OLLIVIER, personal communication) that the rate of genetic improvement in French pigs is about 5 index points per year, and that an increase of one index point is worth about 1.50 francs per pig.

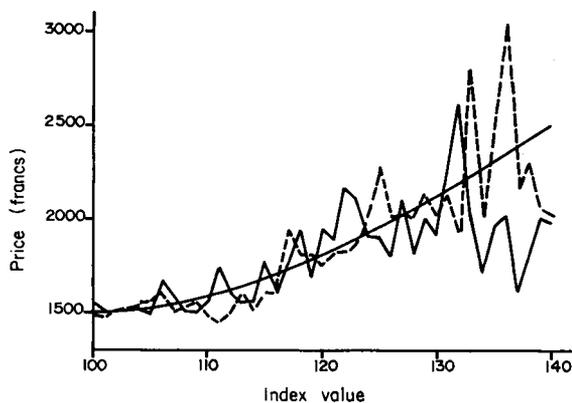


FIG. 1. — Median prices of boars in relation to index of breeding value
(Les prix des verrats en fonction de leur valeur génétique)

—— Landrace - - - - Large White
smooth curve = approximative equation
courbe régulière = l'équation d'approximation

Thus if we take our time unit as being 6 months, G is 2.5 index points per period, and $B = 1.5$ francs per point. Each boar bought could produce about 200 piglets reared per period ($P = 200$) though this may be somewhat high on average. However, if we accept these values we have :

$$\text{PBG} = 750 \text{ francs per period}$$

Then since the price of an average boar is 1 500 francs, $Q_o = 2$. Since $S = (I - 100)/2.5$, the approximate price function (6) can be rewritten as :

$$1\ 500 + 12.5 S + 3.125 S^2$$

so that

$$k(S) = 1 + S/120 + S^2/480$$

On the other hand, the theory developed in the previous section predicts that competition should tend to produce a price function in which :

$$\begin{aligned} k(S) &= \left(1 + \frac{S}{\sqrt{2Q_o}}\right)^2 \\ &= (1 + \frac{1}{2} S)^2 \end{aligned}$$

This would give

$$\begin{aligned} C &= 1\ 500 \left(1 + \frac{1}{2} \left(\frac{I - 100}{2.5}\right)\right)^2 \\ &= 60 (I - 95)^2 \end{aligned}$$

The result would be extremely high prices for high-index boars. For example, the price of a boar with an index of 140 would be 121 500 francs under this system rather than the observed value of 2 500 francs. It is thus clear that if the above estimates are correct, French pig producers are able to buy genetically superior boars very cheaply.

Assuming that boar prices can be adequately described by equation (6) :

$$U = a + \frac{1}{2}(T + I) - S + 2 \left(1 + \frac{S}{120} + \frac{S^2}{480}\right)/T$$

and therefore :

$$\frac{\partial U}{\partial S} = -1 + \frac{2 + S}{120 T}$$

Since S has an upper limit of about 20, $\frac{\partial U}{\partial S}$ is negative unless T is less than one sixth and therefore it pays to buy the best possible sires. With the existing price structure producers could spend more to buy the best available sires in competition with A.I. centres and breeders. Also,

$$\frac{\partial U}{\partial T} = \frac{1}{2} - \frac{2}{T^2} \left(1 + \frac{S}{120} + \frac{S^2}{480}\right)$$

Supposing a producer buys boars with an index value of 140, so that $S = 16$,

$$\frac{\partial U}{\partial T} = \frac{1}{2} - \frac{10}{3T^2}$$

giving a stationary value when $T^2 = 20/3$ or $T = 2.58$. Thus, using the above

estimates, a producer would be advised to buy the best available sires and to use them for about 2.5 periods or 15 months before replacing them with the best sires then available. Boar purchase costs would then be about 1 000 francs per boar used per period.

The above discussion suggests that genetically superior boars are cheap relative to average boars. It is also of interest to see whether average boars are cheap or expensive relative to their genetic value using the discounted gene flow procedure for evaluation of a purchased sire.

An average sire costs 1 500 francs and it is easily checked that the optimum usage period is two time units or 12 months. We shall then assume that such a boar has progeny born on average when he is 2.5 time units old. If it is assumed that the effects of sow selection in the producer's population on lag can be neglected the lag will be $2l_B G$ when average boars are bought so that $R = 2l_B G$. Then the net present value of an average boar is :

$$\text{N.P.V.} = r^{Y+1} l_B J 750 - 1\ 500$$

Letting $r = 0.95$ corresponding to a discount rate of 10 per cent per annum or 5 per cent per time unit, and letting $Y = 1$ we have :

$$\text{N.P.V.} = 680 l_B J - 1\ 500.$$

Sows are normally one year old at first litter with subsequent litters at 6 month intervals. We may then use the sow parity distribution reported by LEGAULT, DAGORN and TASTU (1975) to find the average age of sows. The approximate age distribution of farrowing sows would then be :

Age (6 month units) :	2	3	4	5	6	7	8	9
Percentage	27	21	17	13	9	6	4	3

The average age of dams would then be 4.05 time units. Since the average of sires is 2.5 time units this gives $l_B = 3.275$ time units so that :

$$\text{N.P.V.} = 2\ 227 J - 1\ 500$$

Using a discount rate of 5 per cent for a 6 month time unit, $r = 1/1.05$ and then :

$$1/J = 1 - \frac{1}{2}(.27r^2 + .21r^3 + .17r^4 + \dots + .04r^8 + .03r^9)$$

from which $J = 1.7010$. Therefore :

$$\text{N.P.V.} = 2\ 288 \text{ francs}$$

That is, an average boar gives a sum of discounted returns in his descendants of about 3 788 francs while costing 1 500 francs, leaving a net value of about 2 288 francs to the buyer. Since average sires are profitable and better sires have been shown to be cheap relative to average sires, superior sires are clearly very profitable, provided that the assumptions on which this analysis has been based are correct.

Discussion

The analysis presented in this paper clearly has a number of limitations. For the most part, attention has been concentrated on a steady state situation, whereas in practice sire buyers will often, perhaps nearly always, have to make

purchase decisions in a non-equilibrium context. In principle this problem might be approached by use of dynamic programming methods, but it is not obvious how to set up an appropriately general model. In addition, the present investigation has considered sire purchase from a particular nucleus, whereas buyers usually have a choice between several, sometimes very many, sources of breeding males. If these are all improving at the same rate, differences between nucleus populations play the same role in the theory as differences within populations, and in principle this should lead to the same relation between price and breeding value between and within nucleus populations. If this is not so, a sire of given breeding value will be cheaper in some studs than in others, and the cheaper sires should be favoured. If the rate of genetic gain is not the same in every nucleus, then in the long run, that nucleus making most rapid progress should become dominant and supply the great majority of sires. However, if there are other nucleus populations which are initially superior, it may be best to buy sires from these in the early stages, buying from the nucleus making fastest progress only when it has surpassed its competitors, though this will clearly depend on prices.

The problem will be more complicated if the nucleus is producing breeding stock for commercial populations which do not have the same economic weights in all cases. Each sire would then have several breeding values, different ones for different potential buyers, and this would probably greatly complicate the effect of price on breeding policies. In fact, although it is hoped that the results of this study will prove useful, there is obviously much more to be done to clarify the sire purchasing problem.

It is interesting to note that when the theory is applied to purchase of French boars the actual decisions made by buyers seem far from optimal. This would not be surprising, since the optimal policy is not easy to see, especially since it involves varying time of use of a boar in relation to his breeding value and price. However, there are other possibilities for the discrepancy. One would be that genetic gains are not being obtained at the rate assumed, or at least they are not perceived as being obtained at that rate by sire buyers. This might be because of disagreement about the genetic changes occurring or because of disagreement about the economic value of the genetic changes which are taking place.

In addition, other economic factors may modify the price relationship. For example, in a time when the market for produce is poor, the incomes of producers will be low and they may lack the finance necessary to pay very high prices for superior sires. Again, if there is an excess supply of sires, there may be insufficient competition for the best ones to force those wishing to buy them to pay high prices.

A factor which has not been accounted for in the above analysis is the possible resale value of sires. It is unlikely that average sires would bring good prices when resold by producers after use for a few time units, but sires of very high breeding value used only for a short time by a producer could conceivably have a resale value of the same order of magnitude as less valuable sires bought directly from the nucleus, so that the net purchase price of highly selected sires would be less than the actual purchase price. The complicating factor is that the price on resale would decline with length of time the sire was used, and so if this factor were important, the optimum number of times to use a superior sire would be reduced. This aspect may deserve further study.

It has also been assumed that inbreeding may be neglected. In our context, this implies a producing population which is large enough so that if a superior

sire is to be used many times it will be possible to avoid mating him to his daughters. When producing populations are too small for this the maximum use of superior sires will have to be less, and we would expect the price differential for high breeding value to be reduced.

However, although the present analysis has several limitations, it does suggest a method of planning sire purchase and use, and indicates the possibility that good sires may be rather cheap for the benefits they confer in some cases at least.

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Résumé

L'analyse des politiques d'achat de géniteurs

L'achat d'un géniteur sélectionné conduit à des gains de productivité dans sa descendance. Une expression de la valeur nette actualisée des recettes qui en résultent est établie. Cette expression est utile pour juger de la valeur d'un achat donné mais elle n'est pas suffisante pour déterminer une politique générale d'achat de géniteurs. Dans ce but, une analyse est faite de la valeur d'un achat de mâle en provenance d'un noyau de sélection qui réalise des gains génétiques constants quand le retard de la population commerciale sur le noyau s'est stabilisé en fonction d'une politique donnée. Cela conduit à un critère pour décider de la durée d'utilisation d'un mâle, qui dépend de son prix, de sa valeur génétique et du progrès génétique dans le noyau. La politique d'achat optimale dépend de la relation entre le prix et la valeur génétique du mâle. La concurrence entre les acheteurs peut modifier la relation entre le prix et la valeur génétique, et aboutir à une relation telle que tous les pères ont la même valeur nette actualisée à condition qu'ils soient convenablement utilisés. Il est montré que, dans ce cas, le prix est une fonction de second degré de la valeur génétique.

Les prix payés par les producteurs pour les verrats vendus aux enchères dans les stations françaises de contrôle individuel sont liés à la valeur de l'indice (I) par la relation approchée :

$$\text{Prix (francs)} = 1\,500 + 5(I - 100) + 0,5(I - 100)^2$$

Pour une telle fonction, la meilleure politique, pour le producteur, consiste à acheter des verrats à haut indice (environ 140) et à les remplacer après une utilisation de 15 mois. La valeur nette actualisée d'un verrat moyen est estimée à 2 000 francs.

Appendix

Definitions of symbols

- a* Age of sire one period before first progeny born.
A Genetic lag between nucleus and base.
B Economic worth of one unit of breeding value.

C	Purchase cost of a sire.
d	Discount rate.
$D_{MB}(d_{FB})$	Genetic superiority of sires (dams) used in base.
e'	Row vector (1 0 . . . 0 0), transpose of e .
f_j	Genetic contribution of j th female age group to progeny.
F	Female gene flow matrix.
G	Genetic gain per time unit.
I	Unit matrix.
J	Leading term of matrix $(I - rF)^{-1}$.
$k(S)$	Cost of a sire with superiority S relative to the cost of an average sire.
$l_{MB}(l_{FB})$	Average age of sires (dams) used in base.
n	Number of sires bought per period.
N	Number of sires in use at any one time.
P	Number of progeny per sire born in one period.
Q	Standardised purchase cost of a sire = C/PBG .
r	Discount factor = $1/(1 + d)$.
R	Sire's breeding value relative to base mean.
S	Standardised genetic superiority of a sire = D_{MB}/G .
T	Maximum number of times a sire is used.
U	Standardised variable cost = $V/NPBG$.
V	Variable cost of sire buying policy.
W	Survival rate of sires from one period to the next.
Y	Age of progeny at which returns are realised.

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